January 31, 1884.

THE PRESIDENT in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

Pursuant to notice Anton De Bary, Carl Gegenbaur, Leopold Kronecker, Rudolph Virchow, and Gustav Wiedemann were severally balloted for and elected Foreign Members of the Society.

The following Papers were read:—

I. "Determination of the Vertical and Lateral Pressures of Granular Substances." By ISAAC ROBERTS, F.G.S., F.R.A.S. Communicated by J. F. BATEMAN, F.R.S. Received November 6, 1883.

PART I.—*Wheat and Peas.*

The investigations which I have the honour to submit in this communication have been undertaken to furnish data to engineers and others who are concerned with the erection of structures which have to sustain pressure upon floors and retaining walls, and also to further the objects of science in a field that is believed to be new.

Store-houses have been erected, both in this country and in America, which consist of rectangular cells, called *bins*, and are filled with grain to a height of 50 feet and upwards above the ground.

Diligent inquiries have been made for any scientific data or rules by which the necessary strength of the walls and floors of such structures could be computed, or, for instance, what would be the vertical and lateral pressure of a column, say of wheat, 14 feet square at its base and top, and 60 feet in height. It was generally assumed to be something less than the pressure of a similar column of water, but how much less did not appear to be known either in England or America.

Last year I made experiments to obtain data, by employing models of square and hexagonal *bins*, particulars of which were communicated in a paper read in Section G, at the meeting of the British Association for the Advancement of Science, held at Southampton. I give briefly, in tabular form, the results.

Table of the Results of Experiments made to ascertain the Vertical Pressure of Wheat stored in Cells or Bins.

Description of cell.	Length of one side.	Diameter of inscribed circle.	Area of bottom.	Height of cell.	Height of maximum pressure.	Mean pressure on bottom.	Weight of wheat in cell when full.	Number of trials made.
	ins.	ins.	ins.	ins.	ins.	lbs.	lbs.	
Square........	7	7	49	36	11	8·5	46·8	17
Hexagon......	4	7	41·57	60	12·5	7	70·2	26
"	$6\frac{7}{8}$	12	122·8	60	24	48	202·8	18
"	12	$20\frac{3}{4}$	374·11	96	36	224	1014	19

It will be observed that all pressure upon the bottom ceases in each cell, at a point not exceeding in height two diameters of its inscribed circle. In the 7-inch square cell, the pressure ceases at the height of 11 inches. In the 7-inch hexagonal cell, it ceases at the height of $12\frac{1}{2}$ inches. In the 12-inch hexagonal cell, it ceases at the height of 24 inches, and in the $20\frac{3}{4}$-inch hexagonal cell, it ceases at the height of 36 inches. So the eighty trials which were made with the four cells point clearly to one conclusion, namely, *that in any cell which has parallel sides, the pressure of wheat upon the bottom ceases when it is charged up to twice the diameter of its inscribed circle.*

The weight of a cubic foot of wheat when filled loosely is 49 lbs., and the weight when shaken and pressed into the measure is 53 lbs. With these data, and those given in the foregoing Table, we may determine the pressure-figure, or form, which the wheat assumes in the cell. Let us take the 12-inch hexagonal cell as the type. Its area is equal to 122·8 square inches. All pressure upon the bottom ceased when the wheat was 24 inches in height, and the mean pressure upon the bottom then was 48 lbs., which is very nearly the weight of 1 cubic foot of wheat when loosely filled. Then 122·8 square inches × 24 inches = 2947·2 cubic inches, but the pressure was only equal to 1728 cubic inches of wheat (1 cubic foot) and, therefore, the pressure-form must be some figure containing approximately 1728 cubic inches of wheat. We have three dimensions of the figure ascertained by experiment, namely, the area of its base, its height, and its cubical capacity, or, 122·8 ins. the base, 24 ins. the height, and 1728 cubic inches the capacity. The figure that meets these conditions is therefore *parabolic*, and I suggest the following formula to determine its cubical capacity and pressure upon any given area :—

Let a = area of cell in feet.
d = diameter of inscribed circle in feet.
c = the constant determined by experiment.
w = weight of wheat in pounds per cubic foot.
p = pressure upon the bottom of cell in pounds.

If we apply this formula to the cell now under consideration, we have—

$$a \times d \times c \times w = p,$$

or
$$\left\{ \frac{122{\cdot}8 \text{ ins.}}{144 \text{ ins.}} \times 1 \times 1{\cdot}03 \times 53 \text{ lbs.} \right\} = 46{\cdot}56 \text{ lbs.}$$

The actual pressure was ascertained by weighing to be 48 lbs., and this was the total mean pressure upon the bottom of the cell, though it was filled with wheat that weighed 202·8 lbs. I now know, and will give the particulars later on, that if this cell had been lengthened to 60 ft. or any other height, the pressure upon the bottom would not have exceeded 48 lbs.

The investigations could not be considered conclusive if left at this stage, since they in a remarkable manner appeared to be contradictory to all our generally received knowledge of the laws governing the flow and pressure of fluids. It was also reasonable to doubt their reliability if applied on a large scale, notwithstanding their close agreement with each other in all the experimental cells. Besides, there was no evidence furnished by the experiments to show what pressure the sides of the cells would have to sustain, although it might be inferred that the side pressure upon a unit of surface, would be less than that upon a similar unit of bottom surface, but how much less, if any, was not indicated. I therefore decided to try the expements on a large scale, and to include in them determinations both of the vertical and lateral pressures. These I now proceed to describe.

Mr. Paul, the well-known corn-merchant of Liverpool, at the suggestion of Mr. Grayson, architect (both of whom took great interest in the inquiry), placed at my disposal part of a warehouse at the Duke's Dock, together with the use of grain, men, and machinery for manipulating with. In one corner of the warehouse I constructed a rectangular *bin*, two sides of which were formed with spruce deal planks, 3 inches in thickness, placed on end, so that they were supported only at the level of each floor, and had a clear bearing in the ground storey, of 9 ft. 3 ins. The other two sides of the *bin* were the walls of the warehouse. On plan the *bin* measured internally 6 ft. 9 ins. by 6 ft. 0 in., and 52 ft. 2 ins. in height, and it contained when filled 2112 cubic feet of wheat, weighing 111,976 lbs.

The weighing apparatus consisted of two graduated levers, one for weighing the vertical, and the other for weighing the lateral pressures.

They were made according to my directions by the well-known machine makers, Messrs. Henry Pooley and Sons. Each lever was capable of weighing with great accuracy from one pound weight to 1800 lbs., advancing one pound at a time, by a sliding weight, which was caused to run along the graduated edge of the lever by the pulling of a cord. The woodcut on page 236 shows the levers and the other adjuncts placed in position for weighing. The upper figure shows the machine for weighing lateral pressures, and the lower figure that for weighing vertical pressures. Both levers were tested by me with standard weights up to 800 lbs., and half a pound weight would clearly move the lever when any part of the 800 lbs. was applied to the machine. I now proceed to describe the methods of weighing.

The bottom of the *bin* was formed 4 feet above the basement floor of the warehouse, and an opening 3 feet square was made in it centrically. Beneath this opening a slide was placed which could be forced through the grain by means of a powerful screw-jack, or be withdrawn from beneath the grain by means of ropes and pulley-blocks as might be required. Beneath this slide were grooves into which a frame could be inserted, and when so placed it would be concentric with the 3-ft. aperture already described in the bottom of the *bin*. Three frames were made to fit into these grooves, with apertures of 1 ft. by 1 ft., 2 ft. by 2 ft., and 3 ft. by 3 ft. respectively. Beneath any one of the apertures when placed in position in the grooves, was the board which formed the top of the weighing-machine, and the arrangements were such that a column of grain measuring 1 ft., 4 ft., or 9 ft. superficial area, and of any height, up to 51 ft. 9 ins., could at any time be weighed upon the lever of the machine, without the necessity of emptying the *bin* to insert the *aperture* frames after each weighing had been completed. It was also necessary, in order to insure accuracy, that all pressure should be taken off the machine before proceeding with each successive weighing, and these conditions were attained by opening and closing the slides in manner already referred to.

The weighings of the lateral pressures were accomplished in manner slightly different from those of the bottom or vertical. The lever of the machine was placed at right angles with the side of the *bin*, in the way shown on the figure. The pressure-receiving top of the machine, and its bracket, were suspended by a chain from their common centre of gravity, and when placed in their position for weighing, they just touched (without exerting any pressure) the knife-edged fulcræ of the lever, and a pressure equal to less than one pound weight would always be indicated by the lever rising off its rest.

Three apertures of the respective areas of 1 ft., 4 ft., and 9 ft.,

were employed in weighing the side pressures, and the slide for admitting and stopping the grain between the weighing-machine and the *bin* was similar to that described for bottom pressure, only that it ran on edge instead of horizontally. The details are shown on the figure.

The method adopted in weighing both the vertical and lateral pressures was the following: On three sides of the *bin*, painted gauge boards were affixed and marked in feet, and each foot was subdivided into spaces of 3 inches. These boards extended from the top of the weighing-machines to the top of the *bin*, and the quantity of grain which I desired to weigh was put into the *bin* and carefully levelled at the desired height, by the aid of these marks. Then three or more weighings were taken at that level. Afterwards the height of the grain in the *bin* was increased to the next desired stage, and then levelled and weighed three times, or more, and so on, stage by stage, until the *bin* was filled.

Before the grain was allowed to come in contact with the weighing-machines, care was taken always to place an excess of weight upon the levers, and this was reduced to indicate the pressure by moving the sliding weight by means of the cord along the graduated edge of the lever. The reduction of the weight on the weighing-machine was therefore steady and gradual.

I had not proceeded far with the process of weighing, when I observed a constantly recurring stiffness in the rising of the levers off their rests. They would rise, say one-fiftieth of an inch off the rest, and there remain for an indefinite length of time without the least further movement; but as 1 lb. weight after another was removed off the lever, it gradually rose until the balance was finally overcome, when the lever would rise and touch the check which prevented further motion upwards. The difference between the first movement and the final rise of the lever frequently equalled one-half the whole pressure. This condition invariably accompanied every weighing. The machines were carefully examined, but no fault lay there, and I infer that the difference here referred to represents the elastic force of grain when it is subject to pressure, and that this elastic force expends itself before the mass of the grain comes into the state of mobility.

In the following tables, which contain the results of each reliable weighing which was made, I distinguish between the first small movement of the lever and the final movement which represents the grain in motion, by the words *Dormant* and *Active* pressures. I call the pressure *dormant* when the lever has been lifted off its rest to the extent that a beam of light can be seen between the lever and its rest; and the pressure *active*, when the lever has risen about three-fourths of an inch above its rest up to the check.

Table I.—Weighings of the Bottom or Vertical Pressures of Wheat.

Size of the Aperture used=3 ft. 0 in. × 3 ft. 0 in.=9 square feet.

	Wheat 6 ft. high in bin.		Wheat 8 ft. high in bin.		Wheat 8 ft. 9 ins. high in bin.		Wheat 51 ft. 9 ins. high in bin.	
	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.
	..	938						
	..	924						
	..	928						
	..	874						
	1202	1034	1159	924	1146	1003	1191	826
	1150	1024	..	..		..	1182	828
	1119	957	..	..	..	..	1145	832
Mean	1157	954·14	1159	924	1146	1003	1172·67	828·67

Table II.—Weighings of the Bottom or Vertical Pressures of Wheat.

Size of the Aperture used=2 ft. 0 in. × 2 ft. 0 in.=4 square feet.

	Wheat 3 ft. high in bin.		Wheat 4 ft. high in bin.		Wheat 6 ft. high in bin.		Wheat 51 ft. 9 ins. high in bin.	
	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.
	..	..	..	502				
	..	..	..	500	..	..	638	344
	..	457	..	494	..	..	685	370
	..	453	600	500	..	..	702	349
	..	473	585	444	..	494	701	352
Mean	..	461	592·5	488	..	494	681·5	353·75

Table III.—Weighings of the Bottom or Vertical Pressures of Wheat.

Size of the Aperture used=1 ft. 0 in. × 1 ft. 0 in.=1 square foot.

	Wheat 2 ft. 4 ins. high in bin.		Wheat 51 ft. 9 ins. high in bin.	
	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.
	..	50	193	50
	..	50	226	50
	..	50	229	50
Mean	..	50	216	50

Table IV.—Weighings of the Side or Lateral Pressures of Wheat.

Size of the Aperture used=3 ft. 0 in. × 3 ft. 0 in.=9 square feet.

	Wheat 6 ft. high in bin.		Wheat 7 ft. 3 ins. high in bin.		Wheat 9 ft. high in bin.		Wheat 45 ft. 6 ins. high in bin.	
	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.
	272	225						
	279	265						
	..	214	..	..	..	224	367	218
	335	263	385	300	..	227	350	285
	330	235	300	250	..	220	313	250
	261	202	325	259	..	230	365	240
Mean	295·4	234	336·67	269·67	..	255·25	348·75	248·25

Note.—In all the weighings of side pressures the height of the wheat in the bin is measured from the *lower edge* of the apertures respectively.

Table V.—Weighings of the Side or Lateral Pressures of Wheat.

Size of the Aperture used=2 ft. 0 in. × 2 ft. 0 in.=4 square feet.

Wheat 2 ft. high in bin.		Wheat 4 ft. high in bin.		Wheat 6 ft. high in bin.		Wheat 6 ft. 9 ins. high in bin.		Wheat 8 ft. high in bin.		Wheat 45 ft. high in bin.	
Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.
68	43	..	..	..	..	..	..	165	83		
96	38	..	..	..	..	..	..	167	78		
95	40	150	83	160	82	170	80	160	79	238	100
75	39	128	81	150	82	158	87	174	80	206	92
..	..	..	..	..	..	..	..	167	89	195	87
83·5	40	139	82	155	82	164	83·5	166·6	81·8	213	93

Table VI.—Weighings of the Side or Lateral Pressures of Wheat.

Size of the Aperture used=1 ft. 0 in. × 1 ft. 0 in.=1 square foot.

	Wheat 1 ft. high in bin.		Wheat 2 ft. high in bin.		Wheat 4 ft. high in bin.		Wheat 44 ft. 6 ins. in bin.	
	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.
	..	..	14·5	11	..	..	44	10
	8	2	13	6	15	6	47	6
	11	2	16	6	15	6	41	8
Mean	9·5	2	14·5	7·76	15	6	44	8

Table VII.—Weighings of the Bottom or Vertical Pressures of Peas. Size of the Aperture used=3 ft. 0 in. × 3 ft. 0 in.=9 square feet.

	Peas 9 ft. high in bin.		Peas 49 ft. 6 ins. high in bin.	
	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.
	1323 1308	1255 1236	1374 1391	1324 1324
Mean...	1315·5	1245·5	1382·6	1324

Table VIII.—Weighings of the Bottom or Vertical Pressures of Peas. Size of the Aperture used=2 ft. 0 in. × 2 ft. 0 in.=4 square feet.

	Peas 6 ft. high in bin.		Peas 8 ft. 7 ins. high in bin.		Peas 49 ft. 10 ins. high in bin.	
	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.
	723 690	544 597	738 746	601 561	646 726	506 644
Mean...	706·5	570·5	742	581	686	575

Table IX.—Weighings of the Side or Lateral Pressures of Peas. Size of the Aperture used=3 ft. 0 in. × 3 ft. 0 in.=9 square feet.

	Peas 6 ft. high in bin.		Peas 9 ft. high in bin.		Peas 43 ft. 10 ins. high in bin.	
	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.
	193 185	116 119	227 251	167 161	242 239	201 187
Mean...	189	117·5	239	164	240·5	194

Table X.—Weighings of the Side or Lateral Pressures of Peas.

Size of the Aperture used=2 ft. 0 in. × 2 ft. 0 in.=4 square feet.

	Peas 6 ft. high in bin.		Peas 43 ft. 9 ins. high in bin.	
	Dormant pressure in lbs.	Active pressure in lbs.	Dormant pressure in lbs.	Active pressure in lbs.
	97	67	121	89
	90	53	118	89
Mean...	93·5	60	119·5	89

In the experiments made with the model *bins*, which have been already referred to, the bottom of the model was formed by a loose board, which rested upon an ordinary weighing-machine, and it only just touched the lower edges of the boards forming the sides. The weights were removed a pound at a time off the machine, and the pressure of the wheat was indicated by the bottom falling away, and the wheat consequently running out; but in the large *bin* the bottom measured 6 ft. 9 ins. by 6 ft. 0 in., and therefore it was not practicable to weigh the whole of the pressure upon the bottom in the manner adopted with the models. I therefore decided to weigh columns of grain within the bin, having a base of 9 ft., 4 ft., and 1 square foot respectively. These columns were formed of the mass of the grain contained within the *bin*, and being coincident with its centre, were not subject to any friction against the sides, but only to the friction amongst the particles of the grain.

Referring now to Table I, it deals with a column of wheat having a base area of 9 square feet. The mean *dormant* pressure when 6 ft. in height was 1157 lbs.; when 8 ft. in height it was 1159 lbs.; when 8 ft. 9 ins. in height it was 1146 lbs.; and when 51 ft. 9 ins. in height it was only 1172·67 lbs. One of the weighings at 6 ft. in height showed the pressure to be 1202 lbs.; but the greatest pressure indicated with the height or head of 51 ft. 9 ins. was only 1191 lbs.

The *active* pressure is in every case less at the height of 51 ft. 9 ins. than it is at any height below that. At 6 ft. it reached 1034 lbs; but at 51 ft. 9 ins. it did not exceed 832 lbs.

The actual weight of the column of wheat that indicated these pressures was, in the case of 6 ft. in height, 2862 lbs. (=6ft. 0 in. × 9 ft. 0 in. × 53 lbs.); and in the case of 51 ft. 9 ins. in height, 24,685 lbs. =(9 ft. 0 in. × 51 ft. 9 ins. × 53 lbs.).

In Table II the column of wheat had a bottom area of 4 square feet, and with the height of 51 ft. 9 ins. would weigh 10,971 lbs.= (4 ft. 0 in. × 51 ft. 9 ins. × 53 lbs.); but the mean *dormant* pressure upon the bottom was only 681·5 lbs., whilst at only 4 ft. in height the mean *dormant* pressure was 592·5 lbs.

The *active* pressure, as will be seen in this table also, is less when the column of wheat is 51 ft. 9 ins. in height than it is at any lower level.

In Table III the aperture had an area of 1 square foot, and the column of wheat 51 ft. 9 ins. in height, indicated a mean *dormant* pressure of 216 lbs., and in each weighing an *active* pressure of only 50 lbs., even at the height of 2 ft. 4 ins., this amount of active pressure was indicated. The total weight of a column of wheat 1 foot square and 51 ft. 9 ins. in height would be 2742 lbs., but the maximum *dormant* pressure did not exceed 229 lbs., and the *active* pressure was only 50 lbs. If now we formulate the results obtained and recorded in Tables I, II, and III in terms of those obtained by the model *bins* already referred to, we have:—

a=area of column of wheat in square feet.

d=diameter of inscribed circle in feet.

c=constant determined by experiment.

w=weight of wheat in pounds per cubic foot.

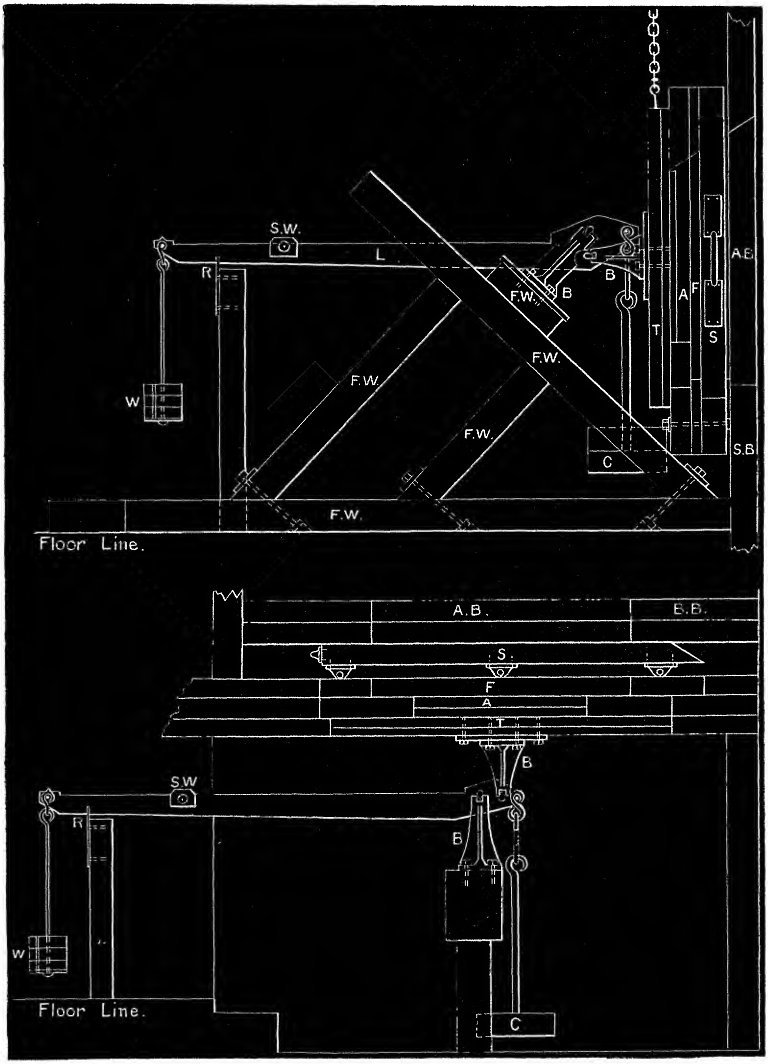
p=pressure upon the bottom.

c is found to be variable within certain limits. With the aperture of 9 square feet, according to Table I, it is 0·84. With the aperture of 4 square feet, according to Table II, it is 1·66, and with the aperture of 1 square foot, according to Table III, it is 4·32.

It, therefore, appears that the apex of the paraboloid which represents the pressure upon 1 square foot area at the centre of a *bin* full of wheat, is higher relatively with its inscribed circle, than it is relatively with the inscribed circle which has its diameter equal to the bottom of the *bin*.

We shall now proceed to consider the lateral or side pressures, and by referring to Tables IV, V, and VI, it will be seen that in each of the weighings, irrespective of the height of the wheat in the *bin*, there is a *dormant* and an *active* pressure both considerably less than the corresponding vertical pressure. With the aperture of 9 square feet the maximum *dormant* pressure of 385 lbs. was attained at the height of 7 ft. 3 ins. above the lower edge of the aperture, and at the height of 45 ft. 6 ins. the maximum pressure was 367 lbs. With the aperture of 4 square feet the maximum *dormant* pressure was 238 lbs., at the height of 45 ft. and at the height of 6 ft. 9 ins. the pressure reached 170 lbs. With the aperture of 1 square foot the maximum *dormant* pressure at 44 ft. 6 ins. in height was 47 lbs.

With the aperture of 2 ft. by 2 ft., and the wheat only 2 ft. in height, that is to say, level with the top edge of the aperture, the *dormant*



pressure was 96 lbs., and when a man whose weight was 170 lbs. stood upon the wheat, close to the aperture, the pressure was 127 lbs., being an increase of 31 lbs. only beyond the pressure of the wheat without the man.

It therefore appears by the weighings of the lateral pressures given in the Tables, that in no case is the maximum *dormant* pressure in a *bin* measuring 6 ft. 9 ins. by 6 ft. on plan, and 45 ft. 6 ins. in

height, greater than 60 lbs. on the square foot of side surface, and that this pressure is nearly uniform between the bottom and 6 ft. off the top, about which point it diminishes gradually to the top.

That the lateral or side pressure cannot be represented, like the bottom pressure, by a parabolic figure, the respective weighings prove, for with the aperture of 1 square foot, and a head of wheat 44 ft. 6 ins. in height, the maximum *dormant* pressure was 47 lbs. With the aperture of 4 square feet, the maximum *dormant* pressure with a head of 45 feet was 59·5 lbs. on each square foot. With the aperture of 9 square feet and a head of 45 ft. 6 ins. the maximum *dormant* pressure was 40·8 lbs. on each square foot.

With the data given in the foregoing pages, we are now in a position to compute both the vertical and lateral pressures, and to determine the distribution of the pressures of the wheat contained in a *bin* of the dimensions given. They are as follows :—

Length of *bin*, 6 ft. 9 ins.; breadth of the *bin*, 6 ft.; height of the wheat, 51 ft. 9 ins.; weight of the wheat, per cubic foot, 53 lbs.; weight of the wheat contained in the *bin*, 111,080 lbs.

Bottom pressure computed by the formula, $a \times d \times c \times w = p$, or the bottom area in square feet into the diameter of the inscribed circle in feet, into the constant, into the weight of wheat in pounds per cubic foot, equal the pressure upon the bottom in pounds. If we substitute the dimensions we shall have (6 ft. 0 in. $\times$ 6 ft. 9 ins.) $=$ 40 ft. 6 in. $\times$ 6 ft. 5 ins. $\times$ 0·84 $\times$ 53 lbs. $=$ 11,569 lbs., the pressure upon the bottom.

The side pressure will be the superficial area in feet of the inside of the walls of the *bin* multiplied by 43 lbs., or, 25 ft. 6 ins. $\times$ 51 ft. 9 ins. $\times$ 43 lbs. $=$ 56,744 lbs., the pressure on the four sides. I adopt 43 lbs., the constant of pressure as shown by the weighings with the 9 square feet aperture, in this calculation, for the reason that it more closely approximates the size of the *bin* than the smaller apertures do. The difference between the sum of the vertical and lateral pressures and the whole of the weight of the wheat contained in the *bin* would be 42,767 lbs., and this would represent the friction of the wheat against the sides. This frictional pressure added to the lateral pressure would represent the weight sustained by the side walls. The wheat is formed into an elastic *plug*, fitting closely to the shape of the *bin*, and each grain is held in sensitive equilibrium by the resultants of the vertical, lateral, and frictional resistances.

If a slide valve were opened in the bottom or side of the *bin* when it is full of wheat, the equilibrium of the grains would be disturbed to the extent of a vertical column having a sectional area somewhat larger than the aperture of the valve, and its height equal to the head of wheat. It would be a vortex column fed mostly by the wheat at the top of the *bin* sliding into it down slopes formed by the angle of

repose (about 30°), and then passing through the mass of the wheat towards the outlet.

The flow of the wheat through an aperture, say of 80 square inches area, can easily be stopped by a board held in the hand against it, though the head may be 50 feet or more.

These statements are not made by inference, for I have proved their accuracy on several occasions. On one I placed some $1\frac{1}{2}$-inch cubes of wood, and a pound weight on the vortex column in a bin containing wheat 37 ft. in height, and they passed through the body of wheat and out at the bottom when 20 ft. of wheat still remained in the *bin*. On another occasion, I am informed by a credible eye-witness, that a cat passed through the mass of wheat by the vortex column and came out alive and uninjured, through the aperture at the bottom of the *bin*. The body of wheat through which it passed exceeded 20 ft. in height.

There is yet one question concerning the lateral pressure to be considered, and that is:—If 43 lbs. equal the amount upon each square foot uniformly from bottom to top in a *bin* which measures 6 ft. 0 in. × 6 ft. 9 ins. × 45 ft. 6 ins., would it be exceeded in a *bin* which measures, say 12 ft. 0 in. × 12 ft. 0 in. × 60 ft. 0 in.? In answering the question, it has already been shown that all lateral pressure ceases at a height not exceeding *two and a-half times* the breadth of the side (see Table IV), and therefore all lateral pressure would cease in the *bin* assumed at the height of 30 ft. (12 ft. 0 in. × $2\frac{1}{2}$); and in order to determine if the lateral pressure would be increased by any increased breadth of the *bin*, I caused one to be constructed within that already described, of the dimensions 3 ft. 0 in. × 3 ft. 0 in. × 12 ft. 0 in., and when it was filled with wheat the maximum *dormant* pressure was 48·55 lbs. upon a square foot. This pressure was also indicated when only 7 ft. of wheat was placed in the *bin*.

This shows that within certain limits the lateral pressure is slightly greater in the small *bin* than it is in the large one; and we may, I think, in practice safely adopt 50 lbs. upon each square foot of side as the constant for the lateral pressure of wheat.

We shall now consider the pressure of peas.

Those selected for the experiment were dry, hard, and round. They weighed, when pressed into the measure, 54 lbs. per cubic foot, and the determinations of the vertical and lateral pressures were made precisely in the same manner, and with the same weighing-machines and *bin* that were used in determining the pressures of wheat. I need not, therefore, repeat any part of the descriptions which have been already given, and will now discuss the tables which embody the results obtained. They are numbered respectively VII, VIII, IX, and X.

Table VII shows the maximum *dormant* pressure upon the bottom

aperture of 9 square feet with the head or height of 49 ft. 6 ins. of peas to be 1391 lbs., and with the head of 9 ft. the pressure of 1323 lbs. With the bottom aperture of 4 square feet (Table VIII) and the head of 49 ft. 10 ins., the maximum *dormant* pressure of 726 lbs. was indicated, and with the head or height of 6 ft., 723 lbs. The apex of the pressure paraboloid is therefore at the height of three diameters of the inscribed circle of the base or aperture, and is greater by one diameter than the wheat paraboloid. The constant (c) for peas is, in the case of the 9 ft. aperture, 0·96; the corresponding constant for wheat being 0·84.

The lateral pressures are given in Tables IX and X, where, with the aperture of 9 square feet and a head of 43 ft. 10 ins. of peas, the maximum *dormant* pressure was 242 lbs., and with the head of 9 ft. 251 lbs. With the aperture of 4 square feet and head of 43 ft. 9 ins. the maximum *dormant* pressure was 121 lbs., and with a head of 6 ft. 97 lbs. It therefore appears that all lateral pressure ceases at a height equal to three times the breadth of the aperture as indicated by the 9 ft., and that the pressure is slightly greater as indicated by the 4 ft. square aperture.

With the 9 ft. it is 27·89 lbs. on each square foot, and with the 4 ft. it is 30·25 lbs. The pressures of wheat corresponding with these (Tables IV and V) were 42·78 lbs. and 59·5 respectively.

The conclusions to be drawn are, that the vertical or bottom pressure of peas is *greater* than that of wheat in the ratio of 0·96 to 0·84, or of 1·73 to 1·66; but that the lateral or side pressure of peas is *less* than that of wheat in the ratio of 27·89 to 42·78, or of 30·25 to 59·5.

The constant (c=0·96) for computing the vertical pressure is safe for the equilibrium of peas, wheat, or any other grain, but the safe permanent sustaining powers of any floors or walls should be at least four times the constant.

For the lateral pressure, 50 lbs. upon each square foot of retaining wall is safe for the equilibrium of all grain; but the safe permanent resistance to pressure in any wall or partition should not be less than four times that, or 200 lbs. upon each foot.

It will have been observed that the *maximum dormant* pressure has been adopted in the formulæ for both the vertical and lateral pressures, though the *active* pressure is that which would cause the breaking down of the structure, and, as shown in the tables, it is considerably less than the *dormant*.

Seeing that the constants for the vertical pressures of both wheat and peas are so near unity, one being 0·84 and the other 0·96, I propose to adopt unity in the formula, which will then stand finally thus :—

$$a \times d \times w = p,$$

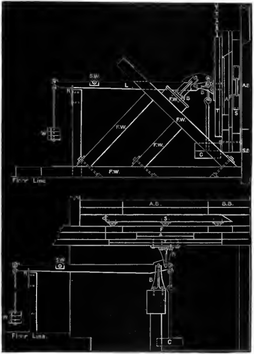
or the area of the bottom of the *bin* in feet multiplied by the diameter of its inscribed circle in feet, and the product multiplied by 54 lbs. (the weight of grain per cubic foot), equal the pressure upon the bottom of the *bin* in pounds, whatever height the grain in the *bin* may be, exceeding two diameters of its inscribed circle.

Following are the weights per cubic foot of various kinds of grain, as determined by myself. The loose and close fillings of the measure are given in each case, but the close or pressed filling is adopted as the element W in the formula.

Weight of grain per cubic feet.	Loosely filled into measure.	Closely filled into measure.
Wheat :—	lbs.	lbs.
Red winter	49	$53\frac{1}{2}$
Bombay	49	53
California	49	53
Walla Walla	46	$50\frac{1}{2}$
Bessarabia	49	53
Peas :—		
American	50	54
Indian corn :—		
White American	$43\frac{1}{2}$	47
Mixed ,,	44	47
Oats :—		
Russian	28	33
Beans :—		
Egyptian	46	50
Barley (English)	39	44

EXPLANATION OF THE FIGURE.

A. Aperture through which the grain is brought into contact with the top of the weighing machine.
B. Brackets to support the levers.
A.B. Apertures in the side and bottom of the *bin*.
B.B. Floor of the *bin*.
S.B. Side of *bin*.
F. Free space between *bin* and apertures.
S. Slide for admitting and shutting off the grain.
T. Top of weighing machine.
R. Rest for lever.
S.W. Sliding weight.
W. Suspended weights.
L. Levers.
C. Counterpoise weights.
F.W. Frame of machine.


S.W.
L
F.W.
B
A.B.
A
F
T
S
W
F.W.
F.W.
F.W.
C
S.B.
F.W.
Floor Line
A.B.
B.B.
S
F
S.W.
B
W
C
Floor Line